

# Black Hole Lensing and Wave Bursts

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## Abstract

It is shown that close to a black hole horizon wave equations have real-valued exponentially time-dependent solutions and to investigate strong gravitational lensing we need to introduce an effective negative cosmological constant between the Schwarzschild and photon spheres. Then exponentially amplified reflected waves from this effective AdS space could explain properties of some gamma ray bursts, fast radio bursts and gravitational waves.

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Gravitational wave (GW) signals [1], gamma ray bursts (GRBs) [2] and fast radio bursts (FRBs) [3] are mysterious transients of extragalactic origin, whose emission mechanisms are still doubtful. Usually these signals are associated with massive black holes (BHs), the most enigmatic objects in the universe. This article explains these unusually strong signals by means of the amplification of the gravitationally strongly lensed waves by BHs. This will explain repeat GWs, GRBs and FRBs detected from the same location and some speculations that these signals could be physically connected.

Weak gravitational lensing is well studied [4], however, wave deflection still remains a difficult subject in the strong gravitational field [5]. Several attempts were made to obtain the adequate lens equations for so called relativistic lensing in Schwarzschild space-time [6].

The most promising focusing agents for any kind of waves are BHs, which are often located at the centers of galaxies and are surrounded by dust clouds. So there are problems of the BH lensing observations, because of spurious radiation and large extinctions of waves by accreting materials, especially of relativistic images, which should be formed much closer to the BHs [7].

In the case of electromagnetic radiation it is expected that, due to the absorptions (larger for smaller wavelength), only the short duration relativistic lensing signals in the forms of radio (largest wavelength) and gamma (most energetic) waves could escape surrounding

BH dust. For GWs, dust clouds and noise do not present an obstacle, also the collinear requirement is less severe (since GWs occur at low frequencies) and the resulting focused region has a relatively large area (because of diffraction). These increase chances to observe several relativistic images of the same GW source, since waves passing close to the BH can loop around it before reaching an observer. Then on the same side next to the source a primary image (formed by weak lensing) and several relativistic images will be visible [7], and unlike the case with the single GRBs and FRBs impulses a distant observer will detect periodic GW signal.

Let us consider waves in the field of a non-rotating, uncharged BH with the line element,

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2(d\theta^2 + \sin^2\theta d\phi^2) , \quad (1)$$

where

$$f(r) = 1 - \frac{2M}{r} . \quad (2)$$

In our system of units ( $c = \hbar = 16\pi G = 1$ ) the mass parameter  $M$  here has the dimension of length. Without losing generality, one can study scalar waves in the background (1), since in the eikonal approximation the polarization tensors are parallel-transported along the null geodesic and in many situations can be regarded to be constant [8].

For a massless scalar particle,  $\Phi(x^\alpha)$ , the wave (Klein-Gordon) equation in a curved space-time has the form:

$$\frac{1}{\sqrt{-g}}\partial_\mu (\sqrt{-g}g^{\mu\nu}\partial_\nu)\Phi(x^\alpha) = 0 . \quad (3)$$

The Schwarzschild space-time (1) is highly symmetric, so in the equation (3) we can separate the variables,

$$\Phi(x^\alpha) \sim \psi(t, r)Y_{lm}(\theta, \phi) , \quad (4)$$

$Y_{lm}(\theta, \phi)$  are spherical harmonics. For the waves with zero angular momentum,  $l = 0$ , the equation (3) gives:

$$[r^2\partial_t^2 - f\partial_r(r^2f\partial_r)]\psi(t, r) = 0 . \quad (5)$$

Let us assume that close to the Schwarzschild horizon,  $r \approx 2M$ , it's possible to separate the variables in (5),

$$\psi(t, r) = \frac{1}{r}T(t)R(r) . \quad (6)$$

Then (5) leads to the system of equations:

$$\begin{aligned} \frac{d^2T}{dt^2} &= CT , \\ f^2\frac{d^2R}{dr^2} + \frac{2Mf}{r^2}\frac{dR}{dr} - \left(C + \frac{2Mf}{r^3}\right)R &= 0 , \end{aligned} \quad (7)$$

where  $C$  is the separation constant. From the last expression it is clear that close to the Schwarzschild horizon the radial wave function should satisfy the condition:  $R(r)|_{f \rightarrow 0} \rightarrow 0$ . Then it is convenient to use the function  $f(r)$  as an independent variable (instead of  $r$ ) and to rewrite the second equation of (7) in the form:

$$f^2(1-f)^4R'' + f(1-f)^3(1-3f)R' - [4M^2C + f(1-f)^3]R = 0 , \quad (8)$$

where primes denote derivatives with respect to  $f$ . If we will look for the solution to this equation in the form:

$$R(f) = \sum_i^{\infty} a_i f^i, \quad (i = 1, \dots, \infty) \quad (9)$$

where  $a_i$  are some constants, then (8) reduces to the algebraic system for the coefficients of  $f^i$ . The first equation of this system ( $i = 1$ ),

$$f(1-f)^3(1-3f)a_1 - [4M^2C + f(1-f)^3]a_1f \approx (1-4M^2C)(a_1f) = 0, \quad (10)$$

shows that the separation constant in (7) is positive,

$$C = 1/4M^2 > 0, \quad (11)$$

and from the first equation of (7) we find the real-valued solution,

$$T(t) = T_0 e^{\pm t/2M}. \quad (12)$$

So in the Schwarzschild coordinates of a distant observer the wave function (6) close to the BH horizon has the form [9]:

$$\psi(t, r) \sim \frac{e^{\pm t/2M}}{r} \sum_i^{\infty} a_i \left(1 - \frac{2M}{r}\right)^i. \quad (13)$$

For the constants we have  $(a_{i+1}/a_i)_{i \rightarrow \infty} \rightarrow 2$  [9], what means that the condition of convergence of the radial wave function (9),  $(a_{i+1}f^{i+1}/a_i f^i)_{i \rightarrow \infty} < 1$ , is satisfied for  $f < 1/2$ , i.e. when  $r < 4M$ .

The real-valued function (13) is very different from the familiar internal [10] and external [11] periodic-in-time solutions ( $\sim e^{i\omega t}$ ) for scalar particles in the field of a BH. Another difference is that in our case the condition  $r \rightarrow 2M$  influences only the spatial component,  $R(r)$ , of the wave function (6) and never the time component,  $T(t)$ . The exponential enhancement (decay) of amplitudes of strongly lensed waves close to a BH horizon leads to the apparent non-conservation of energy from the point of view of a distant observer and can be used to explain observed energetic GWs, GRBs and FRBs.

To study strong lensing we need to investigate isotropic geodesics close to the BH horizon. In general, trajectories of classical particles can be considered as the geometrical-optical limiting case (eikonal approximation) of a wave movement [12]. It is known that the Klein-Gordon wave functions associated with the classical motion,  $\sim e^{iS}$  ( $S$  is the Hamilton principal function), formally obey the relativistic Hamilton-Jacobi (geodesic) equation written for the same system. Indeed, the scalar wave function (4) can be expressed in terms of an amplitude and phase,

$$\Phi = |\Phi| e^{iS}. \quad (14)$$

The relativistic Hamilton's principal function  $S(x^\nu)$ , as usual, can be used in the definition of the 4-velocity field,  $u^\nu \sim D^\nu S$ , where  $D^\nu$  denotes covariant derivatives. If one neglects variations in  $|\Phi|$ , then from (3) follows that the eikonal phase,  $S(x^\nu)$ , in the short wavelength limit,  $g_{\alpha\beta} D^\alpha D^\beta S \rightarrow 0$ , obeys the Hamilton-Jacobi equation,

$$g_{\alpha\beta} u^\alpha u^\beta = 0. \quad (15)$$

However, close to the Schwarzschild horizon variations in the amplitude of the classical scalar wave functions (14) cannot be neglected, since as we have found  $|\Phi(r \rightarrow 2M)| = e^{\pm t/2M}$ , i.e. standard geodesic (gravitational lens) equations should be modified in this region.

In this respect we want to remind the models of classical interpretation of the Schrödinger wave function, where variations of  $|\Phi|$  are taking into account by introduction of so called quantum potential [13, 14]. For example, in the thermodynamic approach [14], the variation of the distribution density (wave amplitude) of a moving particle is modeled by a resistance of the ensemble in the form of the heat flow. For the BH case, appearance of the heat flow close to the Schwarzschild horizon is analogous to the 'firewall' conjecture, the existence of energetic curtain at the event horizon [15], as if BHs have effective quantum 'atmospheres' in the range of  $2M \leq r \leq 3M$  [16].

Using this analogy, in the quasi-classical approximation one can still describe trajectories of particles close to a BH horizon by ordinary geodesic equations, but with extra potential, which takes into account variations of the amplitude (12). Relativistic invariant vacuum energy in the General Relativity is known under the name of cosmological constant,  $\Lambda$ . Solution of the spherically symmetric Einstein equations with the cosmological term is well known, provided that the factor  $f(r)$  is replaced in (1) by the function:

$$F(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2. \quad (16)$$

The constant  $\Lambda$  should be fixed in the way which enables us to use Klein-Gordon equation (7) close to the Schwarzschild horizon and at the same time to cancel apparent energy non-conservation for a distant observer.

Inserting the function (16) into (7), instead of  $f(r)$ , and expanding the solution of the modified equation (8) as the series of  $F(r)$ , the equation (10) takes the form:

$$\begin{aligned} F(1-F)^3(1-3F)a_1 - [4M^2C + F(1-F)^3]a_1F = \\ = [(1-F)^3(1-4F) - 4M^2C]a_1F = 0. \end{aligned} \quad (17)$$

We require validity of this equation at the horizons of  $F(r)$  and  $f(r)$  simultaneously, i.e. at  $F = 0$  and at  $F = -4M^2\Lambda/3$ . This gives

$$C \approx \frac{1}{4M^2}, \quad \frac{\Lambda}{3} \approx -\frac{2}{5M^2}. \quad (18)$$

So the effective cosmological constant  $\Lambda$  is negative, what corresponds to the effective AdS space,

$$F(r) = 1 - \frac{2M}{r} + \frac{2r^2}{5M^2}, \quad (19)$$

for classical particles moving inside the shell between the Schwarzschild and photons spheres,  $2M < r < 3M$ . Let us study isotropic geodesics in this effective AdS space.

It is known that geodesic equations can be obtained using the Lagrangian [8],

$$L(x^\nu) = \frac{1}{2}g_{\alpha\beta}u^\alpha u^\beta, \quad (20)$$

where  $u^\alpha = dx^\alpha/ds$  denotes the 4-velocity and  $ds$  is the proper time. From (19) and (20) we find the expressions for two conserved components of 4-momentum:

$$\begin{aligned}\frac{\partial L(x^\nu)}{\partial u^t} &= u_t F(r) = E, & \left(\theta = \frac{\pi}{2}\right) \\ \frac{\partial L(x^\nu)}{\partial u^\phi} &= u_\phi r^2 = J,\end{aligned}\tag{21}$$

where  $E$  and  $J$  denote the waves energy and angular momentum and  $u_t = dt/ds$ ,  $u_\phi = d\phi/ds$  are 4-velocities.

Inserting (21) into the condition

$$2L = F u_t^2 - \frac{u_r^2}{F} - r^2 u_\phi^2 = 0, \quad (u_\theta = 0)\tag{22}$$

which is satisfied for the isotropic geodesics, we find the first integral of the last independent geodesic equation,

$$u_r^2 = E^2 - \frac{J^2}{r^2} F(r) = E^2 - U^2, \tag{23}$$

where  $u_r = dr/ds$  and  $U^2$  denote the effective potential,

$$U^2 = \frac{J^2}{r^2} \left(1 - \frac{2M}{r} + \frac{2r^2}{5M^2}\right) = U_{Sch}^2 + \frac{2J^2}{5M^2}, \tag{24}$$

which differs from the Schwarzschild potential,  $U_{Sch}^2$ , by the positive constant  $2J^2/5M^2$ . So the maximum of  $U^2$  (which also located on the photon sphere,  $r = 3M$ ) is higher and its zero,

$$r = \frac{2}{\sqrt{-\Lambda}} \sinh \left[ \frac{1}{3} \operatorname{arsinh} \left( 3M\sqrt{-\Lambda} \right) \right] \approx 1.2M. \tag{25}$$

is shifted inside the Schwarzschild sphere,  $r = 2M$  [17].

FIG 1. displays the effective potential in our model (bold blue line) for  $J = M = 1$ . Fare from the BH (where  $\Lambda = 0$ ) the effective potential is done by  $U_{Sch}^2$  and obtains the value (24) inside the photon sphere,  $r < 3M$ . It is clear that a non-radial isotropic geodesics (dashed line) with the energy above or equal to the critical value,  $E_c^2 = J^2/27M^2$ , could cross the photons sphere and fall towards the center of BH following the spiral trajectory [8, 17]. However, according to our model, inside the shell between the Schwarzschild and photon spheres,  $2M < r < 3M$ , waves cross the effective AdS space, where the actual impact factor,  $E^2/J^2 - 2/5M^2$ , changes the sign. In this region the effective potential (24) forms the second higher barrier. So waves could stay in the minimum between two pics of  $U^2(r)$  looping on some bound orbit in the range  $2M < r < 3M$  (FIG. 1). Exact values of the perihelion and aphelion depend on the parameters of the effective potential  $U(r)$  and the initial impact factor.

The waves with decreasing amplitude, according to (12), will lose energy and plunge into singularity. At the same time photons with increasing amplitude will be reflected from the effective AdS space and a distant observer will detect burst-like short signals from the BH edge, mainly in gamma and radio frequencies which could escape surrounding the BH dust clouds.

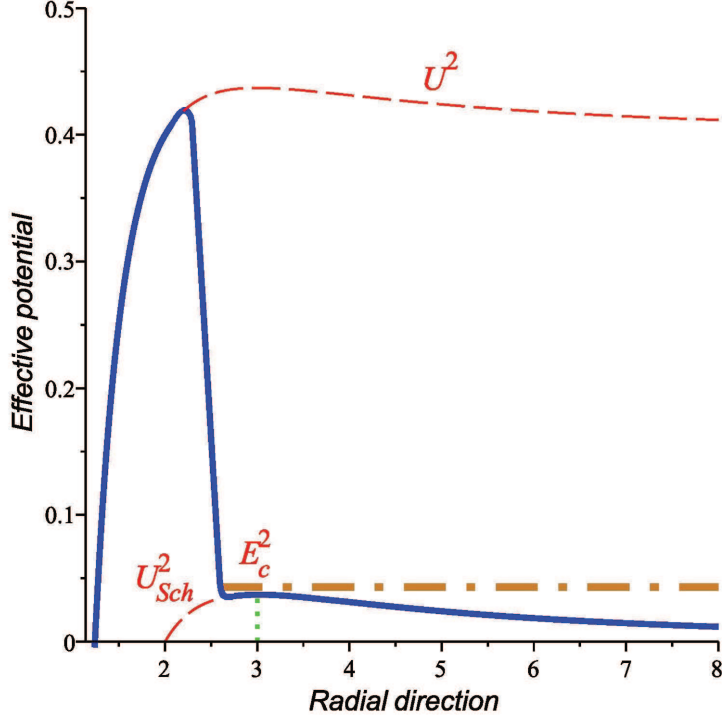


Figure 1: The effective potential for non-radial waves.

In the case of GWs there is chance to observe also relativistic lensing and additionally we need to explore the obtained from (21) and (23) radial equation:

$$\left(\frac{dr}{dt}\right)^2 = F^2(r) \left[ 27M^2 \frac{F(r)}{r^2} - 1 \right]. \quad (26)$$

Inside the shell  $2M < r < 3M$  we can use the approximation  $F(r) \approx 2r^2/5M^2$  and for the inspiraling GWs ( $dr/dt < 0$ ) the equation (26) gives:

$$r \approx \frac{15M^2}{18t + 5M} \approx 3M - 11t, \quad (27)$$

where the integration constant is fixed from the condition  $r(t = 0) = 3M$ . Then from (19) and (21) for the instant frequency of the looping GWs we find:

$$\omega = \frac{d\phi}{dt} = \frac{3\sqrt{3}M}{r^2} F(r) = \frac{11}{10M} + \frac{9}{5M^2} t. \quad (28)$$

When the effective impact parameter changes the sign, spiraling GWs will start deflecting and a distant observer will receive the periodic chirp like signals with the exponential amplitudes (12) and increasing frequency (28). Then the expression for the timing of the resulted strain,

$$h \sim Ae^{t/2M} \sin \omega t + Be^{-t/2M} \cos \omega t, \quad (29)$$

where  $A$  and  $B$  are some constants, can be described by a linear superposition of several quasi-normal modes of perturbed BH horizon by GWs [18] and will imitate the LIGO signals [1].

To conclude, in this paper we have shown that close to a BH horizon wave equations in the Schwarzschild coordinates have the real-valued exponentially time-dependent solutions. While quantum wave functions and isotropic geodesics are continuous below the photons sphere,  $r < 3M$ , for a distant observer receiving signals only from the region,  $r > 2M$ , these exponential enhancement (decay) of amplitudes will be visible as if the gravitationally lensed waves receiving (giving) the energy from (to) the BH. To model isotropic geodesics in the strong gravitational field between the Schwarzschild and photon spheres we have introduced the effective negative cosmological constant with the value that guaranties validity of the eikonal approximation in this region. Then we found that part of the incident waves crossing the effective AdS space below the photon sphere can be amplified and reflected and this mechanism can explain some GWs, GRBs and FRBs.

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